Received: *July 9, 2001,* Accepted: *October 23, 2001*
Revised: *October 10, 2001*

D-brane dynamics and logarithmic superconformal algebras

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Abstract: We construct the consistent supersymmetric extensions of the operators describing the recoil of a D-brane and show that they realize an $\mathcal{N}=1$ logarithmic superconformal algebra. The corresponding supersymmetric vertex operator is related to the action of a twisted superparticle with twist field determined by the angular momentum of the recoiling D-brane and with explicitly broken κ-symmetry. We show that the superconformal completion removes the logarithmic modular divergences that are present in the bosonic string loop scattering amplitudes. These features are all consequences of the relationship that exists in these models between worldsheet rescaling and the time evolution of the D-brane in target space.

Keywords: D-branes, Conformal Field Models in String Theory, Supersymmetry and Duality.

JHEP10(2001)027

Contents

1. Introduction

One of the ways of describing the dynamics of D-branes is to regard them as string solitons. The center of mass zero modes of the soliton break target space translational symmetry. This effect can be described in an open string worldsheet formalism as inducing a deformation of the underlying conformal field theory σ-model [1]. The change of state of the soliton during a scattering event leads to a relevant deformation of the σ-model and causes a change in the conformal field theory background. One surprising aspect of this description is that the quantization of the collective coordinates of the D-brane induces violations of conformal invariance in the form of non-local worldsheet operators, yielding unexpected logarithmic terms [1]. These scaling violations were subsequently shown [2] to be a consequence of the fact that the worldsheet Noether currents associated with spatial translations should be identified with logarithmic operators [3] of the theory. These logarithmic operators correspond to hidden (target space) continuous symmetries [4] related to the collective coordinates. The proper operators describing the recoil effects during the scattering of closed string states off the D-brane background have been constructed and their logarithmic conformal algebra is well-studied [5]–[7].

Logarithmic conformal field theories lie on the border between conformal field theories and generic renormalizable two-dimensional field theories. They are characterized by the appearence of logarithmic terms in the four-point correlation functions of primary operators. The logarithmic divergences are due to the fact that two (or more) primary fields have the same scaling dimensions (modulo integers) and that they can no longer diagonalize the Virasoro generators of the theory [3]. Rather, they mix under conformal transformations and form Jordan cells. While such theories are not unitary, they define realistic models which can still be classified to some extent by conformal data. In certain cases, the logarithmic scaling violations lead to subtleties in their worldsheet renormalization group properties, implying a sort of marginal non-criticality of the theory. This is precisely the case with the recoil deformation operators for string solitons, which are marginally relevant, and are thereby capable of driving the deformed σ-model to a non-trivial fixed point. These operators are described in a semi-classical impulse approximation, appropriate for the non-relativistic dynamics of heavy D-branes, or equivalently to leading tree-level order of string perturbation theory. It is induced by the capture of a closed string state by the D-brane defect. In the worldsheet formalism such a procedure implies [7] the splitting of a closed string state into a pair of open strings, or in other words the approach of a worldsheet bulk operator to the worldsheet boundary. This leads to a bulk-boundary operator product expansion [8].

While the recoil operators are unusual and difficult to handle, they have passed several non-trivial consistency checks. For example, the operators induce a target space dynamics which is equivalent to that derived from the Born-Infeld action, even in the case of non-abelian, multiple D-brane configurations [7]. In this paper we will show that these operators are also consistent with $\mathcal{N} = 1$ supersymmetry. In particular, we will construct supersymmetric partners to the recoil operators and show that they naturally close to a supersymmetric extension of a logarithmic conformal algebra. The original example of an $\mathcal{N} = 1$ logarithmic superconformal field theory is provided by a certain class of supersymmetric Wess-Zumino-Witten models, in which some operator product expansions are found to contain logarithmic terms [2, 9]. Supersymmetric extensions of logarithmic conformal field theories have been subsequently dealt with in generality in [10, 11]. The following construction extends the pair of impulse operators for D-branes to a consistent deformation of the underlying supersymmetric worldsheet σ-model, and at the same time provides one of the first examples of a logarithmic superconformal field theory.

Besides demonstrating the overall consistency of the recoil operators, the incorporation of supersymmetric partners enables us to study the quantum dynamics of the D-brane background in the full setting appropriate to branes in Type II superstring theory. We shall uncover a number of remarkable properties of the $\mathcal{N} = 1$ supermultiplet of recoil operators. For instance, we shall see that in the case of D0-branes a target space supersymmetrization naturally leads to the interpretation of the superspace trajectories described by the impulse operator as those of a certain twisted superparticle [12], with twist field determined by the angular momentum of the recoiling D-particles. A certain class of such spinning superparticle theories possesses the same spectrum as supersymmetric Yang-Mills theory in ten dimensions. The particular classical configurations can be seen to explicitly break the target space

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supersymmetry, which is expected from a time-dependent D-brane background. This gives a novel physical interpretation of the breaking of supersymmetry by moving D-branes.

We shall also see that the logarithmic superconformal structure has dramatic effects on the structure of open string loop amplitudes which describe the target space quantum corrections to the scattering of elementary string states off the D-brane. We will find that the fermionic superpartners remove the logarithmic modular divergences which in the bosonic case arise from string states associated with logarithmic operators of vanishing conformal dimension [2, 7]. While the cancellation of instabilities in the purely bosonic string theory is expected from worldsheet superconformal invariance, in the logarithmic case it is a non-trivial cancellation because the spinor fields also exhibit the same sort of logarithmic scaling violations in their correlation functions. We will see that the stabilization can be attributed to the identification of the worldsheet scale with an evolution parameter in target space that was made initially in [6] to extract the proper logarithmic correlation functions. In the following we will see that this identification alternatively follows from the requirement of cancellation of modular divergences and also from the interpretation of the impulse operator in terms of twisted superparticles with explicitly broken supersymmetry.

The structure of this paper is as follows. In section 2 we describe the $\mathcal{N} = 1$ supersymmetric extension of the general logarithmic conformal algebra. In section 3 we explicitly construct the supersymmetric extensions of the impulse operators and show that they satisfy the structure of a logarithmic superconformal field theory. In section 4 we show that these same operators arise naturally from the standard worldsheet supersymmetric Wilson loops which describe the dynamics of D-branes in the T-dual picture, and also how they appear canonically in a superspace formulation of the problem. In section 5 we analyse the interplay between worldsheet scale transformations and target space time evolution of the D-brane, and show that the impulse operator explicitly breaks super-Galilean invariance. We also describe the relationship to the spacetime supersymmetric Wilson loop and use this to interpret the recoil operator as that of a twisted superparticle in the case of D0-branes. Finally, in section 6 we demonstrate the cancellation of modular divergences in superstring annulus amplitudes and show how it may be most naturally understood in terms of the Zamolodchikov metric corresponding to the superconformal logarithmic operators.

2. Logarithmic superconformal field theories

The Virasoro algebra of a two-dimensional conformal field theory is generated by the worldsheet energy-momentum tensor $T(z)$ with the operator product expansion

$$T(z)\,T(w) = \frac{c/2}{(z-w)^4} + \frac{2}{(z-w)^2}\,T(w) + \frac{1}{z-w}\,\partial_w T(w) + \cdots\,, \qquad (2.1)$$

where c is the central charge of the theory, and an ellipsis always denotes terms in the operator product expansion which are regular as $z \to w$. For a closed surface these relations are accompanied by their anti-holomorphic counterparts, while for an open surface the coordinates z, w are real-valued and parametrize the boundary of the worldsheet. In the following we will be concerned with the latter case corresponding to open strings and so will not write any formulas for the anti-holomorphic sector. We shall always set the worldsheet infrared scale to unity to simplify the formulas which follow.

The simplest logarithmic conformal field theory is characterized by a pair of operators C and D which become degenerate and span a 2×2 Jordan cell of the Virasoro operators. The two operators then form a logarithmic pair and their operator product expansion with the energy-momentum tensor involves a non-trivial mixing [3]

$$\begin{aligned} T(z)\,C(w) &= \frac{\Delta}{(z-w)^2}\,C(w) + \frac{1}{z-w}\,\partial_w C(w) + \cdots\,, \\ T(z)\,D(w) &= \frac{\Delta}{(z-w)^2}\,D(w) + \frac{1}{(z-w)^2}\,C(w) + \frac{1}{z-w}\,\partial_w D(w) + \cdots\,, \end{aligned} \tag{2.2}$$

where Δ is the conformal dimension of the operators determined by the leading logarithmic terms in the conformal blocks of the theory, and an appropriate normalization of the D operator has been chosen. Because of (2.2), a conformal transformation $z \mapsto w(z)$ mixes the logarithmic pair as

$$\begin{pmatrix} C(z) \\ D(z) \end{pmatrix} = \left(\frac{\partial w}{\partial z}\right)^{\begin{pmatrix} \Delta & 0 \\ 1 & \Delta \end{pmatrix}} \begin{pmatrix} C(w) \\ D(w) \end{pmatrix}, \tag{2.3}$$

from which it follows that their two-point functions are given by [3, 4]

$$\begin{aligned} \Big\langle C(z)\,C(w) \Big\rangle &= 0\,, \\ \Big\langle C(z)\,D(w) \Big\rangle &= \frac{\xi}{(z-w)^{2\Delta}}\,, \\ \Big\langle D(z)\,D(w) \Big\rangle &= \frac{1}{(z-w)^{2\Delta}}\Big(-2\xi\ln(z-w) + d\Big)\,, \end{aligned} \tag{2.4}$$

where the constant ξ is fixed by the leading logarithmic divergence of the conformal blocks of the theory and the integration constant d can be changed by the field redefinition $D \mapsto D + (\text{const.})\,C$. The vanishing of the CC correlator in (2.4) is equivalent to the absence of double or higher logarithmic divergences. From these properties it is evident that the operator C behaves similarly to an ordinary primary field of scaling dimension Δ, while the properties of the D operator follow from the formal identification $D = \partial C / \partial \Delta$.

It is straightforward to write down a superconformal extension of the algebra (2.2) [10]. In this paper we shall only deal with $\mathcal{N}=1$ supersymmetry, but, as will become apparent, the extension to the $\mathcal{N}=2$ case is immediate. The $\mathcal{N}=1$ superconformal algebra is generated by the energy-momentum tensor $T(z)$ with the relations (2.1) and an additional generator, the supercurrent $G(z)$, which is the worldsheet superpartner of $T(z)$. It has conformal weight $\frac{3}{2}$ and the operator product expansions

$$\begin{aligned} T(z)\,G(w) &= \frac{3/2}{(z-w)^2}\,G(w)+\frac{1}{z-w}\,\partial_w G(w)+\cdots\,, \\ G(z)\,G(w) &= \frac{\hat{c}}{(z-w)^3}+\frac{2}{z-w}\,T(w)+\cdots\,, \end{aligned} \tag{2.5}$$

where $\hat{c}=2c/3$ is the superconformal central charge. We introduce fermionic fields χ_C and χ_D which are the worldsheet superpartners of the operators C and D, respectively. For simplicity, throughout this article, we shall work only in the Neveu-Schwarz sector of the theory corresponding to the choice of anti-periodic boundary conditions on the worldsheet spinor fields. Then the fields χ_C and χ_D are generated by the operator product expansions

$$G(z)\,\phi(w)=\frac{1/2}{z-w}\,\chi_\phi(w)+\cdots\,, \qquad \phi=C,D\,. \tag{2.6}$$

One can now derive the $\mathcal{N}=1$ supersymmetric completion of the logarithmic conformal algebra (2.2). The pair (C,χ_C) satisfies the standard algebraic relations of a primary superconformal multiplet of dimension Δ, while the additional relations for χ_D can be obtained by differentiating those involving χ_C with the formal identification $\chi_D=\partial\chi_C/\partial\Delta$ [10]. The $\mathcal{N}=1$ logarithmic superconformal algebra is thereby characterized by the operator product expansions (2.2), (2.6), and

$$\begin{aligned} T(z)\,\chi_C(w) &= \frac{\Delta+1/2}{(z-w)^2}\,\chi_C(w)+\frac{1}{z-w}\,\partial_w\chi_C(w)+\cdots\,, \\ T(z)\,\chi_D(w) &= \frac{\Delta+1/2}{(z-w)^2}\,\chi_D(w)+\frac{1}{(z-w)^2}\,\chi_C(w)+\frac{1}{z-w}\,\partial_w\chi_D(w)+\cdots\,, \\ G(z)\,\chi_C(w) &= \frac{\Delta}{(z-w)^2}\,C(w)+\frac{1/2}{z-w}\,\partial_w C(w)+\cdots\,, \\ G(z)\,\chi_D(w) &= \frac{\Delta}{(z-w)^2}\,D(w)+\frac{1}{(z-w)^2}\,C(w)+\frac{1/2}{z-w}\,\partial_w D(w)+\cdots\,. \end{aligned} \tag{2.7}$$

In addition to the Green's functions (2.4), the two-point functions involving the extra fields can also be readily worked out to be [10]

$$\begin{aligned} \Big\langle\phi(z)\,\chi_{\phi'}(w)\Big\rangle &= 0\,, \qquad \phi,\phi'=C,D\,, \\ \Big\langle\chi_C(z)\,\chi_C(w)\Big\rangle &= 0\,, \end{aligned}$$

$$\Big\langle \chi_C(z)\,\chi_D(w)\Big\rangle = \frac{2\Delta\xi}{(z-w)^{2\Delta+1}}\,,$$
$$\Big\langle \chi_D(z)\,\chi_D(w)\Big\rangle = \frac{2}{(z-w)^{2\Delta+1}}\Big(-2\Delta\xi\ln(z-w)+\xi+\Delta d\Big)\,. \tag{2.8}$$

Analogous results can be obtained for the three-point correlation functions of the theory [10, 13]. Four-point functions are obtained in [14] and for dimension zero fields in [15]. For the cases where there are n degenerate fields which span an $n\times n$ Jordan cell, the three-point functions have been recently worked out explicitly in [16]. It is also possible to generalize these results to the case where there is more than one Jordan block. Note that, under the assumption that the logarithmic partner fields are quasi-primary, any such Jordan block implies the existence of a Jordan cell for the identity operator, which has vanishing scaling dimension. Thus, if there exists a Jordan block with $\Delta\neq 0$, then there are automatically at least two Jordan blocks for the logarithmic conformal field theory.

3. Supersymmetric impulse operators for moving D-branes

We will now derive the appropriate supersymmetric vertex operator describing the recoil of a D-brane and show that it naturally satisfies a logarithmic superconformal algebra. For simplicity, we shall deal only with the case of 0-branes, but the extensions to generic p-branes are straightforward. At tree-level in open string perturbation theory, such a configuration is described by the harmonic string coordinates $x^\mu=(x^0,x^i)$ which map a disc Σ into flat 1+9 dimensional spacetime with the Dirichlet boundary conditions $x^i|_{\partial\Sigma}=0$ along the transverse directions to the brane and Neumann ones $\partial_\perp x^0|_{\partial\Sigma}=0$ along the D0-brane worldline. Here $\partial_\perp$ denotes the normal derivative at the boundary of the string worldsheet Σ which is parametrized by the periodic coordinate $\tau\in[0,1]$. The bulk of Σ is parametrized by coordinates σ^α, $\alpha=1,2$. The bosonic vertex operator describing the motion of the brane is given by [17]

$$\begin{aligned} V_{\rm D}^{\rm bos} &= \exp\left(-\frac{1}{2\pi\alpha'}\int\limits_\Sigma d^2\sigma\;\eta^{\alpha\beta}\,\partial_\alpha\left[Y_i\Big(x^0(\sigma)\Big)\,\partial_\beta x^i(\sigma)\right]\right)\\ &= \exp\left(-\frac{1}{2\pi\alpha'}\int\limits_0^1 d\tau\;Y_i\Big(x^0(\tau)\Big)\,\partial_\perp x^i(\tau)\right)\,, \end{aligned} \tag{3.1}$$

where α' is the string slope, $\partial_\alpha=\partial/\partial\sigma^\alpha$, and $Y_i(x^0)=\delta_{ij}\,Y^j(x^0)$ describes the trajectory of the D0-brane as it moves in spacetime.

The recoil of a heavy D-brane due to the scattering of closed string states may be described in an impulse approximation by inserting appropriate factors of the usual Heaviside function $\Theta(x^0)$ into (3.1). This describes a non-relativistic 0-brane which begins moving at time $x^0 = 0$ from the initial position y_i with a constant velocity u_i. The appropriate trajectory is given by the operator [6]

$$Y_i(x^0) = y_i\, C_\epsilon(x^0) + u_i\, D_\epsilon(x^0)\,, \tag{3.2}$$

where we have introduced the operators

$$C_\epsilon(x^0) = \alpha'\,\epsilon\,\Theta_\epsilon(x^0)\,, \qquad D_\epsilon(x^0) = x^0\,\Theta_\epsilon(x^0)\,, \tag{3.3}$$

with $\Theta_\epsilon(x^0)$ the regulated step function which is defined by the Fourier integral transformation

$$\Theta_\epsilon(x^0) = \frac{1}{2\pi i}\int\limits_{-\infty}^{\infty}\frac{d\omega}{\omega - i\epsilon}\;\mathrm{e}^{\,i\omega x^0}\,. \tag{3.4}$$

This integral representation is needed to make the Heaviside function well-defined as an *operator*. In the limit $\epsilon \to 0^+$, it reduces via the residue theorem to the usual step function. The operator $C_\epsilon(x^0)$ is required in (3.2) by scale invariance. Note that the center of mass coordinate y_i appears with a factor of $\epsilon \to 0^+$, so that the first operator in (3.2) represents a small uncertainty in the initial position of the D-brane induced by stringy effects [6]. The pair of fields (3.3) are interpreted as functions of the coordinate z on the upper complex half-plane, which is identified with the boundary variable τ in (3.1). This interpretation is possible because the boundary vertex operator (3.1) is a total derivative and so can be thought of as a bulk deformation of the underlying free bosonic conformal σ-model on Σ (in the conformal gauge). The impulse operator (3.1), (3.2) then describes the appropriate change of state of the D-brane background because it has non-vanishing matrix elements between different string states. It can be thought of as generating the action of the Poincaré group on the 0-brane, with y_i parametrizing translations and u_i parametrizing boosts in the transverse directions.

By using the representation (3.4) and the fact that the tachyon vertex operator $\mathrm{e}^{\,i\omega x^0}$ has conformal dimension $\alpha'\omega^2/2$, it can be shown [6] that the operators (3.3) form a degenerate pair which generate a logarithmic conformal algebra (2.2) with conformal dimension $\Delta = \Delta_\epsilon$, where

$$\Delta_\epsilon = -\frac{\alpha'\epsilon^2}{2}\,. \tag{3.5}$$

The total dimension of the impulse operator (3.1), (3.2) is $h_\epsilon = 1 + \Delta_\epsilon$, and so for $\epsilon \neq 0$ it describes a relevant deformation of the underlying worldsheet conformal σ-model. The existence of such a deformation implies that the resulting string theory is slightly non-critical and leads to the change of state of the D-brane background.

The two-point functions of the operators (3.3) can be computed explicitly to be [6]

$$\begin{aligned}
\Big\langle C_\epsilon(z)\, C_\epsilon(w)\Big\rangle &= \frac{1}{4\pi}\sqrt{\frac{(\alpha')^3}{\epsilon^2\ln\Lambda}}\,\left[\frac{\sqrt{\pi}}{2}\;{}_1F_1\left(\frac12\,,\,\frac12\,;4\epsilon^2\alpha'\ln(z-w)\right)-\right.\\
&\qquad\left. -\,2\sqrt{\epsilon^2\alpha'\ln(z-w)}\;{}_1F_1\left(1\,,\,\frac32\,;4\epsilon^2\alpha'\ln(z-w)\right)\right]\;,\\
\Big\langle C_\epsilon(z)\, D_\epsilon(w)\Big\rangle &= \frac{1}{4\pi\epsilon^3}\sqrt{\frac{1}{2\alpha'\ln\Lambda}}\,\left[\frac{\sqrt{\pi}}{8}\;{}_1F_1\left(\frac12\,,\,-\frac12\,;4\epsilon^2\alpha'\ln(z-w)\right)\right.\\
&\qquad\left. +\,\frac{16}{3}\Big(\epsilon^2\alpha'\ln(z-w)\Big)^{3/2}\;{}_1F_1\left(2\,,\,\frac52\,;4\epsilon^2\alpha'\ln(z-w)\right)\right],\\
\Big\langle D_\epsilon(z)\, D_\epsilon(w)\Big\rangle &= \frac{1}{\epsilon^2\alpha'}\Big\langle C_\epsilon(z)\, D_\epsilon(w)\Big\rangle\,,
\end{aligned} \tag{3.6}$$

where $\Lambda\to 0$ is the worldsheet ultraviolet cutoff which arises from the short-distance propagator

$$\lim_{z\to w}\Big\langle x^0(z)\,x^0(w)\Big\rangle = -2\alpha'\ln\Lambda\,. \tag{3.7}$$

Here we have used the standard bulk Green's function in the upper half-plane, as the effects of worldsheet boundaries will not be relevant for the ensuing analysis.[1] This is again justified by the bulk form of the vertex operator (3.1), and indeed it can be shown that using the full expression for the propagator on the disc does not alter any results [7]. It is then straightforward to see [6] that in the correlated limit $\epsilon,\Lambda\to 0^+$, with

$$\frac{1}{\epsilon^2} = -2\alpha'\ln\Lambda\;, \tag{3.8}$$

the correlators (3.6) reduce at order ϵ^2 to the canonical two-point correlation functions (2.4) of the logarithmic conformal algebra, with conformal dimension (3.5) and the normalization constants

$$\xi = \frac{\pi^{3/2}\,\alpha'}{2}\,,\qquad d = d_\epsilon = \frac{\pi^{3/2}}{2\epsilon^2}\,. \tag{3.9}$$

Note that the singular behaviour of the constant d_ϵ in (3.9) is not harmful, because it can be removed by considering instead the connected correlation functions of the theory [6].

We will now derive the $\mathcal{N}=1$ supersymmetric completion of the impulse operator (3.1), (3.2). For this, we introduce 2×2 Dirac matrices ρ^α, $\alpha=1,2$, and real two-component Majorana fermion fields ψ^μ which are the worldsheet superpartners

[1] Boundary effects in logarithmic conformal field theories have been analysed in [7, 18].

of the string embedding fields x^μ. A convenient basis for the worldsheet spinors is given by

$$\rho^1 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \qquad \rho^2 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} , \tag{3.10}$$

in which the fermion fields decompose as

$$\psi^\mu = \begin{pmatrix} \psi_-^\mu \\ \psi_+^\mu \end{pmatrix} . \tag{3.11}$$

The fields (3.11) obey the boundary conditions $\psi_+^\mu|_{\partial\Sigma} = \pm\,\psi_-^\mu|_{\partial\Sigma}$, where the sign depends on whether they belong to the Ramond or Neveu-Schwarz sector of the worldsheet theory [19]. The global worldsheet supersymmetry is determined by the supercharge $\mathcal{Q}$ which generates the infinitesimal $\mathcal{N}=1$ supersymmetry transformations [19]

$$\begin{aligned} \Big[\mathcal{Q}\,,\,x^\mu\Big] &= \psi^\mu\ , \\ \Big\{\mathcal{Q}\,,\,\psi^\mu\Big\} &= -i\,\rho^\alpha\,\partial_\alpha x^\mu\ . \end{aligned} \tag{3.12}$$

The fermionic fields $\psi_+(z)$ have conformal dimension 1/2, and from (3.12) it follows that the superpartner of the tachyon vertex operator $\mathrm{e}^{\,i\omega x^0}$ is $\sqrt{\alpha'}\,\omega\,\psi_+^0\ \mathrm{e}^{\,i\omega x^0}$, so that in the Neveu-Schwarz sector we may write

$$G(z)\ \mathrm{e}^{\,i\omega x^0(w)} = \frac{\sqrt{\alpha'}\,\omega/2}{z-w}\,\psi_+^0(w)\ \mathrm{e}^{\,i\omega x^0(w)} + \cdots\ . \tag{3.13}$$

In what follows it will be important to note the factor of $\sqrt{\alpha'}\,\omega$ that appears in the supersymmetry transformation (3.13). Because of it, and the fact that the tachyon vertex operator has conformal dimension $\alpha'\omega^2/2$, the inverse transformation is given by

$$G(z)\,\psi_+^0(w)\ \mathrm{e}^{\,i\omega x^0(w)} = \frac{\sqrt{\alpha'}\,\omega/2}{(z-w)^2}\ \mathrm{e}^{\,i\omega x^0(w)} + \frac{i/2\sqrt{\alpha'}}{z-w}\,\Big(\partial_w x^0(w)\Big)\ \mathrm{e}^{\,i\omega x^0(w)} + \cdots\ . \tag{3.14}$$

To compute the superpartners of the logarithmic pair (3.3), we use (3.4) and (3.13) to write

$$G(z)\,C_\epsilon(w) = \frac{\epsilon\,(\alpha')^{3/2}/4\pi i}{z-w}\,\psi_+^0(w)\int\limits_{-\infty}^{\infty}\frac{d\omega}{\omega-i\epsilon}\,\Big[(\omega-i\epsilon)+i\epsilon\Big]\ \mathrm{e}^{\,i\omega x^0(w)} + \cdots\ . \tag{3.15}$$

In the first term of the integrand in (3.15) there is no pole and so after contour integration it vanishes. Formally it is a delta functional $\delta(x^0(w))$ which we neglect since we are interested here in only the asymptotic time-dependence of string solitons. Then, only the second term contributes, and comparing with (2.6) we find

$$\chi_{C_\epsilon}(x^0,\psi^0) = i\,\epsilon\,C_\epsilon(x^0)\,\psi_+^0\ . \tag{3.16}$$

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Similarly, we have

$$G(z)\,D_\epsilon(w)=-\frac{\sqrt{\alpha'}/4\pi}{z-w}\,\psi_+^0(w)\int\limits_{-\infty}^{\infty}\frac{d\omega}{(\omega-i\epsilon)^2}\,\Big[(\omega-i\epsilon)+i\epsilon\Big]\;\mathrm{e}^{\,i\omega x^0(w)}+\cdots\,, \tag{3.17}$$

which using (2.6) gives

$$\chi_{D_\epsilon}(x^0,\psi^0)=i\,\left(\epsilon\,D_\epsilon(x^0)-\frac{1}{\epsilon\,\alpha'}\,C_\epsilon(x^0)\right)\,\psi_+^0\,. \tag{3.18}$$

The operators (3.16) and (3.18) have conformal dimension $\Delta_\epsilon+1/2$.

It is straightforward to now check that the remaining relations of the $\mathcal{N}=1$ logarithmic superconformal algebra are satisfied. By using (2.2), (3.5), (3.16) and (3.18), it is easy to verify the first two operator product expansions of (2.7) in this case. For the operator products with the fermionic supercurrent, we use in addition the Fourier integral (3.4) along with (3.14) to get

$$\begin{aligned}
G(z)\,\chi_{C_\epsilon}(w)&=-\frac{\epsilon^2(\alpha')^{3/2}/4\pi i}{(z-w)^2}\int\limits_{-\infty}^{\infty}\frac{d\omega}{\omega-i\epsilon}\,\Big[(\omega-i\epsilon)+i\epsilon\Big]\,\mathrm{e}^{\,i\omega x^0(w)}+\\
&\quad+\frac{1}{z-w}\,\partial_w\chi_{C_\epsilon}(w)+\cdots\\
&=-\frac{\sqrt{\alpha'}\,\epsilon^2/2}{(z-w)^2}\,C_\epsilon(w)+\frac{1}{z-w}\,\partial_w\chi_{C_\epsilon}(w)+\cdots\,,\\
G(z)\,\chi_{D_\epsilon}(w)&=-\frac{\sqrt{\alpha'}/4\pi}{(z-w)^2}\int\limits_{-\infty}^{\infty}\frac{d\omega}{\omega-i\epsilon}\,\left[\Big((\omega-i\epsilon)+i\epsilon\Big)+\frac{i\epsilon}{\omega-i\epsilon}\,\Big((\omega-i\epsilon)+i\epsilon\Big)\right]\times\\
&\quad\times\,\mathrm{e}^{\,i\omega x^0(w)}+\frac{1}{z-w}\,\partial_w\chi_{D_\epsilon}(w)+\cdots\\
&=\frac{1/\sqrt{\alpha'}}{(z-w)^2}\,\left(C_\epsilon(w)-\frac{\alpha'\epsilon^2}{2}\,D_\epsilon(w)\right)+\frac{1}{z-w}\,\partial_w\chi_{D_\epsilon}(w)+\cdots\,,
\end{aligned} \tag{3.19}$$

which also agree with (2.7) in this case.

For the two-point correlation functions (2.8), we use the fermionic Green's function in the upper half-plane,

$$\Big\langle\psi_+^0(z)\,\psi_+^0(w)\Big\rangle=\frac{1}{z-w}\,, \tag{3.20}$$

and the fact that bosonic and fermionic field correlators factorize from each other in the free superconformal σ-model on Σ. The first set of relations in (2.8) are then satisfied in this case because $\langle\psi_+^0(z)\rangle=0$. The second relation holds to order ϵ^4 since $\Delta_\epsilon\propto\epsilon^2$ and $\langle C_\epsilon(z)C_\epsilon(w)\rangle=0$ to order ϵ^2. For the remaining correlators, we use (2.4), (3.16), (3.18), (3.20) and factorization to compute

$$\Big\langle \chi_{C_\epsilon}(z)\,\chi_{D_\epsilon}(w)\Big\rangle = -\frac{\epsilon^2\xi}{(z-w)^{2\Delta_\epsilon+1}}\,,$$
$$\Big\langle \chi_{D_\epsilon}(z)\,\chi_{D_\epsilon}(w)\Big\rangle = \frac{1}{(z-w)^{2\Delta_\epsilon+1}}\left[\frac{2\xi}{\alpha'} - \epsilon^2\Big(-2\xi\ln(z-w)+d_\epsilon\Big)\right], \tag{3.21}$$

which upon using (3.5) are also seen to agree with (2.8). Therefore, the supersymmetric extensions (3.16) and (3.18) of the impulse operators (3.3) give precisely the right combinations of operators that generate the full algebraic structure of a logarithmic superconformal field theory. This yields a non-trivial realization of the supersymmetric completion of the previous section 2, and illustrates the overall consistency of the impulse operators describing the dynamics of D-branes in closed string scattering states.

4. Superspace formalism

We will now derive the explicit form of the supersymmetric extension of the impulse vertex operator (3.1), (3.2). For this, we consider the Wilson loop operator

$$W[A] = \exp i\oint_{\partial\Sigma} A_\mu(x)\ dx^\mu = \exp i\int_0^1 d\tau\ \dot{x}^\mu(\tau)\,A_\mu\Big(x(\tau)\Big)\,, \tag{4.1}$$

where A_μ is a U(1) gauge field in ten dimensions, and $\dot{x}^\mu(\tau) = dx^\mu(\tau)/d\tau$. T-duality maps the operator (4.1) onto the vertex operator (3.1) for a moving D-brane by the rule $\partial_\alpha x^i \mapsto i\,\eta^{\beta\gamma}\,\epsilon_{\alpha\beta}\,\partial_\gamma x^i$ and the resulting replacement of Neumann boundary conditions for x^i with Dirichlet ones [20]. The spatial components of the Chan-Paton gauge field map onto the brane trajectory as $A_i = Y_i/2\pi\alpha'$, while the temporal component A_0 becomes a U(1) gauge field on the D-particle worldline.

The minimal $\mathcal{N}=1$ worldsheet supersymmetric extension of the operator (4.1) is given by

$$\mathcal{W}[A,\psi] = W[A]\ \exp\left(-\frac{1}{2}\int_0^1 d\tau\ F_{\mu\nu}\,\overline{\psi}^\mu\,\rho^1\,\psi^\nu\right)\,, \tag{4.2}$$

where $F_{\mu\nu}$ is the corresponding gauge field strength tensor. For the recoil trajectory (3.2), an elementary computation using the contour integration techniques outlined in the previous section gives $F_{ij}=0$ and

$$F_{0i}(x^0) = \frac{\delta A_i(x^0)}{\delta x^0} = \frac{i}{2\pi\alpha'}\left[y_i\,\epsilon\,C_\epsilon(x^0) + u_i\left(\epsilon\,D_\epsilon(x^0) - \frac{1}{\epsilon\,\alpha'}\,C_\epsilon(x^0)\right)\right]. \tag{4.3}$$

This shows that, in the T-dual Neumann picture, the canonical supersymmetric extension of the U(1) Wilson loop operator (4.2) yields *precisely* the couplings to the operators χ_{C_ϵ} and χ_{D_ϵ} that were computed in the previous section from the supersymmetric completion of the worldsheet logarithmic conformal algebra.

T-duality acts on the worldsheet fermion fields (3.11) by reversing the sign of their right-moving components ψ_-^μ. By using (4.2), (4.3) we may thereby write down the supersymmetric extension of the impulse operator for moving D0-branes,

$$V_{\rm D}^{\rm susy} = \exp\Biggl(-\frac{1}{2\pi\alpha'}\int\limits_0^1 d\tau\,\Biggl\{\Bigl[y_i\,C_\epsilon\Bigl(x^0(\tau)\Bigr)+u_i\,D_\epsilon\Bigl(x^0(\tau)\Bigr)\Bigr]\,\partial_\perp x^i(\tau)+ + \Bigl[y_i\,\chi_{C_\epsilon}\Bigl(x^0(\tau)\,,\,\psi^0(\tau)\Bigr)+u_i\,\chi_{D_\epsilon}\Bigl(x^0(\tau)\,,\,\psi^0(\tau)\Bigr)\Bigr]\,\psi^i(\tau)\Biggr\}\Biggr)\,, \qquad (4.4)$$

where we have dropped the $\pm$ subscripts on the fermion fields in (3.11), and the logarithmic superconformal operators in (4.4) are given by (3.3), (3.16) and (3.18). The vertex operator (4.4) can be expressed in a more compact form which makes its supersymmetry manifest. For this, we extend the disc Σ to an $\mathcal{N}=1$ super-Riemann surface $\hat\Sigma$ with coordinates $(Z,\bar Z)=(z,\theta,\bar z,\bar\theta\,)$, where θ is a complex Grassmann variable, and with corresponding superspace covariant derivatives $\mathcal{D}_Z=\partial_\theta+\theta\,\partial_z$. Given a bosonic field $\phi(z)$ with superpartner $\chi_\phi(z)$, we introduce the chiral worldsheet superfields

$$\Phi_\phi(z,\theta)=\phi(z)+\theta\,\chi_\phi(z)\,, \qquad (4.5)$$

and correspondingly we make the embedding space of the superstring an $\mathcal{N}=1$ superspace with chiral scalar superfields $X^i(z,\theta)=x^i(z)+\theta\,\psi^i(z)$. Then the impulse operator (4.4) can be written in a manifestly supersymmetric form in terms of superspace quantities as

$$V_{\rm D}^{\rm susy}=\exp\left[-\frac{1}{2\pi\alpha'}\oint\limits_{\partial\hat\Sigma} d\tau\ d\theta\ \Bigl(y_i\,\Phi_{C_\epsilon}(\tau,\theta)+u_i\,\Phi_{D_\epsilon}(\tau,\theta)\Bigr)\,\mathcal{D}_\perp X^i(\tau,\theta)\right]\,, \qquad (4.6)$$

where in (4.6) the Grassmann coordinate θ is real.

In fact, the algebraic relations of the logarithmic superconformal algebra can be most elegantly expressed in superspace notation. For this, we introduce the superstress tensor $\mathcal{T}(Z)=G(z)+\theta\,T(z)$, and define the quantities $Z_{12}=z_1-z_2-\theta_1\theta_2$ and $\theta_{12}=\theta_1-\theta_2$ corresponding to a pair of holomorphic superspace coordinates $Z_1=(z_1,\theta_1)$ and $Z_2=(z_2,\theta_2)$. Then the operator product expansions (2.1), (2.2) and (2.5)–(2.7) can also be written in terms of superspace quantities as

$$\mathcal{T}(Z_1)\,\mathcal{T}(Z_2)=\frac{\hat c/4}{(Z_{12})^3}+\frac{3\theta_{12}/2}{(Z_{12})^2}\,\mathcal{T}(Z_2)+\frac{1/2}{Z_{12}}\,\mathcal{D}_{Z_2}\mathcal{T}(Z_2)+\frac{\theta_{12}}{Z_{12}}\,\partial_{z_2}\mathcal{T}(Z_2)+\cdots\,,$$

$$
\begin{aligned}
\mathcal{T}(Z_1)\,\Phi_C(Z_2) &= \frac{\theta_{12}\,\Delta/2}{(Z_{12})^2}\,\Phi_C(Z_2) + \frac{1/2}{Z_{12}}\,\mathcal{D}_{Z_2}\Phi_C(Z_2) + \frac{\theta_{12}}{Z_{12}}\,\partial_{z_2}\Phi_C(Z_2) + \cdots\,, \\
\mathcal{T}(Z_1)\,\Phi_D(Z_2) &= \frac{\theta_{12}\,\Delta/2}{(Z_{12})^2}\,\Phi_D(Z_2) + \frac{\theta_{12}/2}{(Z_{12})^2}\,\Phi_C(Z_2) + \\
&\quad + \frac{1/2}{Z_{12}}\,\mathcal{D}_{Z_2}\Phi_D(Z_2) + \frac{\theta_{12}}{Z_{12}}\,\partial_{z_2}\Phi_D(Z_2) + \cdots\,,
\end{aligned}
\tag{4.7}
$$

while the two-point functions (2.8) may be expressed as

$$
\begin{aligned}
\Big\langle \Phi_C(Z_1)\,\Phi_C(Z_2)\Big\rangle &= 0\,, \\
\Big\langle \Phi_C(Z_1)\,\Phi_D(Z_2)\Big\rangle &= \frac{\xi}{(Z_{12})^{2\Delta}}\,, \\
\Big\langle \Phi_D(Z_1)\,\Phi_D(Z_2)\Big\rangle &= \frac{1}{(Z_{12})^{2\Delta}}\Big(-2\xi\,\ln Z_{12} + d\Big)\,.
\end{aligned}
\tag{4.8}
$$

This superspace formalism also generalizes to the construction of higher-order correlation functions which are built from appropriate coordinate invariants of the supergroup $OSp(1|2)$ [10]. It emphasizes how the impulse operator (4.6), and the ensuing logarithmic algebra (4.7), (4.8), is the natural supersymmetrization of the recoil operators for D-branes.

5. Target space formalism

Let us now describe the target space properties of the logarithmic superconformal algebra that we have derived. A worldsheet finite-size scaling

$$
\Lambda ~\longmapsto~ \Lambda' = \Lambda~\mathrm{e}^{-t/\sqrt{\alpha'}}
\tag{5.1}
$$

induces from (3.8) a transformation of the target space regularization parameter,

$$
\epsilon ~\longmapsto~ \epsilon' = \epsilon + \epsilon^3\,t\,\sqrt{\alpha'} + O(\epsilon^5)\,.
\tag{5.2}
$$

By using (3.9) and the ensuing scale dependence of the correlation functions (2.4) we may then infer the transformation rules

$$
C_{\epsilon'} = C_\epsilon\,, \qquad D_{\epsilon'} = D_\epsilon - \frac{t}{\sqrt{\alpha'}}\,C_\epsilon
\tag{5.3}
$$

to order ϵ^2. It follows that, in order to maintain conformal invariance, the σ-model coupling constants in (4.4) must transform as $y_i \mapsto y_i + (t/\sqrt{\alpha'}\,)\,u_i$, $u_i \mapsto u_i$, and thus a worldsheet scale transformation leads to a Galilean boost of the D-brane in target space.

However, by using (3.16), (3.18), (5.2) and (5.3), we see that the superconformal partners of the logarithmic operators are scale-invariant to order ϵ^2,

$$
\chi_{C_{\epsilon'}} = \chi_{C_\epsilon}\,, \qquad \chi_{D_{\epsilon'}} = \chi_{D_\epsilon}\,.
\tag{5.4}
$$

The invariance property (5.4) can also be deduced from the scale independence to order ϵ^2 of the two-point correlators (2.8), in which the scale dependent constant d_ϵ appears only in the invariant combination $\Delta_\epsilon d_\epsilon \sim O(\epsilon^0)$. This means that the operator (4.4) describes the evolution of the D0-brane in target space with respect to only the ordinary, bosonic Galilean group. In other words, if we introduce a superspace and worldsheet superfields as in (4.5), then a worldsheet scale transformation in the present case acts only on the bosonic part of the superspace. This property is of course very particular to the explicit scale dependence of the recoil superpartners (3.18) in the logarithmic superconformal algebra.

The fact that the super-Galilean group is not represented in the non-relativistic dynamics of D-branes is merely a reflection of the fact that the motion of the brane explicitly breaks target space supersymmetry. Indeed, while the deformed σ-model that we have been working with possesses $\mathcal{N}=1$ *worldsheet* supersymmetry, it is only after the appropriate sum over worldsheet spin structures and the GSO projection that it has the possibility of possessing spacetime supersymmetry. To understand better the breaking of target space supersymmetry within the present formalism, we now appeal to an explicit spacetime supersymmetrization of the Wilson loop operator (4.1). This will produce a Green-Schwarz representation of the spacetime supersymmetric impulse operator in the dual Neumann picture, and also yield a physical interpretation of the supersymmetric vertex operator (4.4).

For this, we regard the Chan-Paton gauge field A_μ as the first component of the ten-dimensional $\mathcal{N}=1$ Maxwell supermultiplet. Its superpartner is therefore a Majorana-Weyl fermion field λ with 32 real components. We introduce Dirac matrices Γ_μ in 1+9 dimensions, and define $\Gamma_{\mu\nu}=\frac{1}{2}[\Gamma_\mu,\Gamma_\nu]$. The loop parametrization $x^\mu(\tau)$ has superpartner $\vartheta(\tau)$ which couples to the photino field λ. Then, the spacetime supersymmetric extension of (4.1) is given by the finite supersymmetry transformation

$$\mathsf{W}[A,\lambda]=\exp\left(\int_0^1 d\tau~\overline{\vartheta}(\tau)\,\mathsf{Q}\right)~W[A]~\exp\left(-\int_0^1 d\tau~\overline{\vartheta}(\tau)\,\mathsf{Q}\right)\,, \tag{5.5}$$

where the supercharge Q generates the infinitesimal $\mathcal{N}=1$ supersymmetry transformations [19]

$$\begin{aligned}\Big[\mathsf{Q}\,,\,A_\mu\Big]&=\frac{i}{2}\,\Gamma_\mu\,\lambda\;,\\ \Big\{\mathsf{Q}\,,\,\lambda\Big\}&=-\frac{1}{4}\,\Gamma_{\mu\nu}\,F^{\mu\nu}\;,\\ \Big[\mathsf{Q}\,,\,x^\mu\Big]&=\frac{i}{4}\,\Gamma^\mu\,\vartheta\;,\\ \Big\{\mathsf{Q}\,,\,\vartheta\Big\}&=4\,.\end{aligned} \tag{5.6}$$

By using the Baker-Campbell-Hausdorff formula, the supersymmetric Wilson loop (5.5) thereby admits an expansion

$$\mathsf{W}[A,\lambda] = \exp i\int_0^1 d\tau\,\left(\dot{x}^\mu\,A_\mu + \frac{i}{4}\,A_\mu\,\overline{\vartheta}\,\Gamma^\mu\,\dot{\vartheta} + \right. \\ \left. + \frac{i}{2}\,\dot{x}^\mu\,\overline{\vartheta}\,\Gamma_\mu\,\lambda + \frac{i}{16}\,\dot{x}^\mu F^{\nu\lambda}\,\overline{\vartheta}\,\Gamma_\mu\,\Gamma_{\nu\lambda}\,\vartheta + \cdots\right), \tag{5.7}$$

where the ellipsis in (5.7) denotes contributions from higher-order fluctuation modes of the fields.

To identify the ten-dimensional supermultiplet which is T-dual to the world-sheet recoil supermultiplet of (4.4), we use the supersymmetry algebra (5.6) to get $\Gamma_i\,\lambda = \frac{1}{2}\,F_{0i}(x^0)\,\Gamma^0\,\vartheta$, with $F_{0i}(x^0)$ given by (4.3). We then find that the target space supermultiplet describing the recoil of a D0-brane is given by the dimensionally reduced supersymmetric Yang-Mills fields

$$A_i(x^0) = \frac{1}{2\pi\alpha'}\,\Big(y_i\,C_\epsilon(x^0) + u_i\,D_\epsilon(x^0)\Big)\,, \\ \lambda(x^0,\vartheta) = \frac{1}{36\pi\alpha'}\,\Gamma^i\,\Big(y_i\,\chi_{C_\epsilon}(x^0,\Gamma^0\,\vartheta) + u_i\,\chi_{D_\epsilon}(x^0,\Gamma^0\,\vartheta)\Big)\,. \tag{5.8}$$

Therefore, the logarithmic superconformal partners to the basic recoil operators also arise naturally in the T-dual Green-Schwarz formalism.

By using (5.7) and (5.8) we can now lend a physical interpretation to the supersymmetric impulse operator. For simplicity, we shall neglect the stringy fluctuations in the center of mass coordinates of the D-brane and take $y_i = 0$. We consider only the long-time dynamics of the string soliton, i.e. we take $x^0 > 0$ and effectively set the Heaviside function $\Theta_\epsilon(x^0)$ to unity everywhere. We will also choose the gauge $A_0(x) = 0$. The bosonic part of the Maxwell supermultiplet of course describes the free, non-relativistic geodesic motion of the D0-brane in flat space. To see what sort of particle kinematics is represented by the full supermultiplet, we substitute $A_i = u_i\,x^0/2\pi\alpha'$ and $\lambda = \not{u}\;\Gamma^0\,\vartheta/36\pi\alpha'$ into (5.7), where $\not{u} = u_i\,\Gamma^i$, and we have again ignored stringy $O(\epsilon)$ uncertainties in position and velocity. Note that, generally, the fermionic operator (3.18) also induces a velocity-dependent stringy contribution to the phase space uncertainty principle in the sense described in [6]. This is reminescent of the energy-dependent smearings that were found in [7]. Heuristically, this identical stringy smearing of position and velocity is responsible for the violation of super-Galilean invariance in (5.4).

With these substitutions we find $\mathsf{W}[A,\lambda] = \mathrm{e}^{\,i\,\mathsf{S}/2\pi\alpha'}$, where

$$\mathsf{S} = \int_0^1 d\tau\,\left(\dot{x}^i\,u_i\,x^0 + \frac{i}{4}\,x^0\,\overline{\vartheta}\,\not{u}\;\dot{\vartheta} + \frac{i}{36}\,\dot{x}^0\,\overline{\vartheta}\,\not{u}\;\vartheta - \frac{i}{4}\,\dot{x}^i\,u_i\,\overline{\vartheta}\,\Gamma^0\,\vartheta + \right. \\ \left. + \frac{i}{32}\,\dot{x}^0\,\overline{\vartheta}\,\left[\Gamma^0\,,\,\not{u}\right]\,\vartheta + \frac{i}{32}\,\dot{\not{x}}\,\overline{\vartheta}\,\left[\Gamma^0\,,\,\not{u}\right]\,\vartheta + \cdots\right) \tag{5.9}$$

can be interpreted as the action of a certain kind of superparticle in the $\mathcal{N}=1$ superspace spanned by the coordinates $(x^i, \vartheta, \overline{\vartheta}\,)$ and with worldline parametrized by the loop coordinate τ. To identify the superparticle type, we will first simplify the last four terms in (5.9). For this, we note that in ten spacetime dimensions the Dirac matrices are taken in a Majorana representation, so that Γ^0 is antisymmetric while Γ^i, $i=1,\dots,9$, are symmetric matrices [21]. We also treat $\vartheta, \dot{\vartheta}$ as an anticommuting pair of variables in the action S. Then, it is easy to check that the third term in (5.9) vanishes, because via an integration by parts it can be written as

$$-\frac{i}{36}\int\limits_0^1 d\tau\ x^0\ \left(\dot{\vartheta}^\top\,\Gamma^0\,\not{u}\ \vartheta+\vartheta^\top\,\Gamma^0\,\not{u}\ \dot{\vartheta}\right)=0\,, \tag{5.10}$$

where we have used the Dirac algebra to write $\Gamma^0\,\not{u} = -\not{u}\,\Gamma^0$. In a similar way one readily checks that the fourth and fifth terms in (5.9) are zero. By the same techniques one finds that the last term is non-vanishing, and after some algebra it can be expressed in the form $\frac{i}{4}\int_0^1 d\tau\ \vartheta^\top\,x^j\,u^i\,\Gamma_{ij}\,\dot{\vartheta}$. The action (5.9) can therefore be written as

$$\mathsf{S}=\int\limits_0^1 d\tau\ \left[p_i\left(\dot{x}^i+i\,\overline{\vartheta}\,\Gamma^i\,\dot{\vartheta}\right)-i\,\ell^\top\,\dot{\vartheta}+\cdots\right], \tag{5.11}$$

where

$$p_i=u_i\,x^0\,, \qquad \ell=x^i\,u^j\,\Gamma_{ij}\,\vartheta\,, \tag{5.12}$$

and we have rescaled the worldline spinor fields $\vartheta\mapsto 2\vartheta$.

The action (5.11) is, modulo mass-shell constraints, that of a twisted superparticle [12], which admits a manifestly covariant quantization. The first term is the standard non-relativistic superparticle action, while the inclusion of the fermionic field ℓ modifies the canonically conjugate momentum to ϑ as $\pi_\vartheta=\not{p}\,\vartheta-\ell$. Note that the quantity p_i in (5.12) is the expected momentum of the uniformly moving D-particle, while ℓ is proportional to its angular momentum. In the present case $p_\mu\,p^\mu\neq 0$, so that the supersymmetric impulse operator describes a *massive*, non-relativistic twisted superparticle. The twist in fermionic momentum π_ϑ vanishes if there is no angular momentum, for instance if the D-particle recoils in the direction of scattering. The equations of motion which follow from the action (5.11), (5.12) can be written as

$$\dot{x}^0=u_i\,\dot{x}^i=\not{u}\ \dot{\vartheta}=0\,, \tag{5.13}$$

which imply that x^0 and the components of x^i and ϑ along the direction of motion are independent of the proper time τ. In general the remaining components of x^μ and ϑ

are τ-dependent. These classical configurations agree with the interpretation of the worldsheet zero mode of the field x^0 as the target space time and also of the uniform motion of the D-particles. In particular, the Galilean trajectory $x^i(\tau) = y^i(\tau) + u^i\, x^0$, appropriate for the kinematics of a heavy D0-brane, solves (5.13) provided that the component of the vector $y^i(\tau)$ along the direction of recoil is independent of the worldline coordinate τ.

There are some important differences in the present case from the standard superparticle kinematics. The action (5.11) generically possesses a fermionic κ-symmetry defined by the transformations

$$\begin{aligned} \delta_\kappa \vartheta &= \not{p}\,\kappa\,, \\ \delta_\kappa \ell &= 2\,p_i\,p^i\,\kappa\,, \\ \delta_\kappa x^i &= i\,\kappa\,\not{p}\,\Gamma^i\,\vartheta\,, \end{aligned} \tag{5.14}$$

where κ is an infinitesimal Grassmann spinor parameter. It is also generically invariant under a twisted $\mathcal{N}=2$ super-Poincaré symmetry [12]. However, the choices (5.12) break these supersymmetries, which is expected because the D-brane motion induces a non-trivial vacuum energy. The configurations (5.12) of course arise from the geodesic bosonic paths in the non-relativistic limit $u_i \ll 1$, or equivalently in the limit of heavy BPS mass for the D-particles, which is the appropriate limit to describe the tree-level dynamics here. The Galilean solutions of (5.13) described above explicitly break the κ-symmetry (5.14).

Therefore, we see that the supersymmetric completion of the impulse operator (for weakly-coupled strings) describes the dynamics of a twisted supersymmetric D-particle in the non-relativistic limit, with a gauge-fixing that breaks its target space supersymmetries. In turn, this broken supersymmetry implies that the vertex operator (4.4) does not generate the action of the super-Poincaré group on the brane, and consequently the super-D-particle does not evolve in target space according to super-Galilean transformations [22]. The structure of the worldsheet logarithmic superconformal algebra is such that these spacetime properties of D-brane dynamics are enforced by the impulse operators.

6. Modular behaviour

In the case of string solitons, the non-trivial mixing between the logarithmic C_ϵ and D_ϵ operators leads to logarithmic modular divergences in bosonic annulus amplitudes, and it is associated with the lack of unitarity of the low-energy effective theory in which quantum D-brane excitations are neglected [2, 6, 7]. We now examine how these features are modified in the presence of the logarithmic $\mathcal{N}=1$ superconformal pair. For this, we consider the open superstring propagator between two scattering states $|\mathcal{E}_\alpha\rangle$ and $|\mathcal{E}_\beta\rangle$,

$$\triangle_{\alpha\beta} = \langle\mathcal{E}_\alpha|\frac{1}{L_0-1/2}|\mathcal{E}_\beta\rangle = -\int\limits_{\mathcal{F}} \frac{dq}{q}\,\langle\mathcal{E}_\alpha|q^{L_0-1/2}|\mathcal{E}_\beta\rangle\,, \tag{6.1}$$

where the Virasoro operator L_0 is defined through the Laurent expansion of the energy-momentum tensor $T(z) = \sum_n L_n\,z^{-n-2}$, and the factor of $\frac{1}{2}$ is the normal ordering intercept in the Neveu-Schwarz sector. Here $q = \mathrm{e}^{\,2\pi i\tau}$, with τ the modular parameter of the worldsheet strip separating the two states $|\mathcal{E}_\alpha\rangle$ and $|\mathcal{E}_\beta\rangle$, and $\mathcal{F}$ is a fundamental modular domain of the complex plane. We shall ignore the superconformal ghosts, whose contributions would not affect the qualitative results which follow.

For the purely bosonic string, divergent contributions to the modular integral would come from a discrete subspace of string states of vanishing conformal dimension corresponding to the spectrum of linearized fluctuations in the soliton background [1, 2, 7]. Since in the present case these are precisely the states associated with the logarithmic recoil operators, we should analyse carefully their contributions to the propagators (6.1). We introduce the highest weight states $|\phi\rangle = \phi(0)|0\rangle$, $\phi = C_\epsilon, D_\epsilon, \chi_{C_\epsilon}, \chi_{D_\epsilon}$, with the understanding that the $\partial_\perp x^i$ and ψ^i parts of the vertex operator (4.4) are included. This has the overall effect of replacing Δ_ϵ in the bosonic parts of the operator product expansions everywhere by the anomalous dimension $h_\epsilon = 1+\Delta_\epsilon$ of the impulse operator, while in the fermionic parts $\Delta_\epsilon + \frac{1}{2}$ is replaced everywhere by h_ϵ. Using (2.2) and (2.7), the 2×2 Jordan cell decompositions of the bosonic and fermionic Virasoro generators are then given by

$$\begin{aligned} L_0^{\rm b}|C_\epsilon\rangle &= h_\epsilon|C_\epsilon\rangle\,, & L_0^{\rm b}|D_\epsilon\rangle &= h_\epsilon|D_\epsilon\rangle + |C_\epsilon\rangle\,,\\ L_0^{\rm f}|\chi_{C_\epsilon}\rangle &= h_\epsilon|\chi_{C_\epsilon}\rangle\,, & L_0^{\rm f}|\chi_{D_\epsilon}\rangle &= h_\epsilon|\chi_{D_\epsilon}\rangle + |\chi_{C_\epsilon}\rangle\,, \end{aligned} \tag{6.2}$$

where $L_0 = L_0^{\rm b} + L_0^{\rm f}$. Using the factorization of bosonic and fermionic states, in the Jordan blocks spanned by the logarithmic operators we have [23]

$$q^{L_0}\,|C_\epsilon,D_\epsilon\rangle\otimes|\chi_{C_\epsilon},\chi_{D_\epsilon}\rangle = q^{h_\epsilon}\begin{pmatrix}1 & 0\\ \ln q & 1\end{pmatrix}|C_\epsilon,D_\epsilon\rangle\otimes q^{h_\epsilon}\begin{pmatrix}1 & 0\\ \ln q & 1\end{pmatrix}|\chi_{C_\epsilon},\chi_{D_\epsilon}\rangle\,. \tag{6.3}$$

The corresponding expectation value (6.1) in such a state is then given by

$$\triangle_{CD} = -\int\limits_{\mathcal{F}} dq\,q^{2\Delta_\epsilon+1/2}\,\langle C_\epsilon,D_\epsilon|\begin{pmatrix}1 & 0\\ \ln q & 1\end{pmatrix}|C_\epsilon,D_\epsilon\rangle\,\langle\chi_{C_\epsilon},\chi_{D_\epsilon}|\begin{pmatrix}1 & 0\\ \ln q & 1\end{pmatrix}|\chi_{C_\epsilon},\chi_{D_\epsilon}\rangle\,. \tag{6.4}$$

The dangerous region of moduli space is $\mathrm{Im}\,\tau\to+\infty$, in which $q\sim\delta\to0^+$. Using $\Delta_\epsilon = 0$ as $\epsilon\to0^+$, we can easily check that the contributions to the modular integration in (6.4) from this region *vanish*. For instance, the worst behaviour comes from the term in the integrand involving $\sqrt{q}\,(\ln q)^2$, which upon integration over a small strip $\mathcal{F}_\delta$ of width δ produces a factor

$$\int\limits_{\mathcal{F}_\delta} dq\ \sqrt{q}\,(\ln q)^2 \simeq \frac{2}{3}\,\delta^{3/2}\,\left((\ln\delta)^2 - \frac{4}{3}\,\ln\delta + \frac{8}{9}\right)\,, \tag{6.5}$$

which vanishes in the limit $\delta \to 0^+$. Therefore, in quantities involving matrix elements of the string propagator in logarithmic states, the incorporation of worldsheet superconformal partners cancels the modular divergences that are present in the purely bosonic case. It is also straightforward to arrive at this conclusion in the Ramond sector of the superstring theory. Notice that although the explicit calculation above is carried out with respect to the chosen basis (6.2) within the Jordan cell, the same qualitative conclusion is arrived at under any change of basis $|C_\epsilon, D_\epsilon\rangle \to |aC_\epsilon + bD_\epsilon, cC_\epsilon + dD_\epsilon\rangle$. This is because the strip integral (6.5) is the worst behaved one and any change of basis will simply mix it with better behaved modular integrals. Furthermore, physical string scattering amplitudes will involve the superstring propagator with sums over complete sets in an invariant, basis-independent form. Its effect on such physical quantities is therefore independent of the chosen base.

This cancellation of infinities has dramatic consequences for the behaviours of higher genus amplitudes. In the purely bosonic case, where the modular divergences persist, the logarithmic states yield non-trivial contributions to the sum over string states and imply that, to leading order, the genus expansion is dominated by contributions from degenerate Riemann surfaces whose strip sizes become infinitely thin [1, 2, 7]. Such amplitudes can be described in terms of bi-local worldsheet operators and the truncated topological series can be summed to produce a non-trivial probability distribution on the moduli space of running coupling constants of the slightly marginal σ-model [7]. The functional Gaussian distribution has width proportional to $\sqrt{\ln\delta}$, and the string loop divergences are cancelled by a version of the Fischler-Susskind mechanism. However, we see here that this structure disappears completely when one considers the full superstring theory. This means that in the supersymmetric case one has to contend with the full genus expansion of string theory which is not even a Borel summable series. The dominance of pinched annular surfaces, as well as the loss of unitarity due to the logarithmic mixing, can now be understood as merely an artifact of the tachyonic instability of the bosonic string. Once the appropriate superconformal partners to the logarithmic operators are incorporated, the theory is free from divergences, at least at the level of string loop amplitudes. Heuristically, this feature can be understood from the form of the fermionic two-point functions (2.8), which for $\Delta = 0$ reduce to conventional fermionic correlators with no logarithmic scaling violations on the worldsheet. The zero dimension fermion fields, after incorporating the worldsheet superconformal ghost fields, thereby have the usual effect of removing instabilities from the theory.

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7. The Zamolodchikov metric

Another way to understand the effect of the fermionic fields in the recoil problem is through the Zamolodchikov metric in the sector corresponding to the logarithmic states. It is defined by the short-distance two-point functions

$$\mathcal{G}_{\phi\phi'} = \Lambda^{2h} \lim_{z\to w} \Big\langle \phi(z)\,\phi'(w)\Big\rangle\,, \qquad \phi,\phi' = C, D, \chi_C, \chi_D\,, \tag{7.1}$$

and by using (2.4) and (2.8) it can be represented as the 4×4 matrix

$$\mathcal{G} = \begin{pmatrix} 0 & \xi & 0 & 0 \\ \xi & d-2\xi\ln\Lambda & 0 & 0 \\ 0 & 0 & 0 & 2\Delta\xi \\ 0 & 0 & 2\Delta\xi & 2(\xi+\Delta d-2\Delta\xi\ln\Lambda) \end{pmatrix}\,. \tag{7.2}$$

In the upper left bosonic 2×2 block we find a logarithmically divergent term, which may be associated to the logarithmic modular divergences that are present in the bosonic case. On the other hand, in the lower right fermionic 2×2 block we find that the logarithmic divergence generically appears only through the term which is proportional to $\Delta\ln\Lambda$. For the recoil problem, in which the conformal dimension of the operators is correlated with the worldsheet ultraviolet scale through the relations (3.5) and (3.8), this term is a finite constant. Thus, in contrast to its bosonic part, the fermionic part of the Zamolodchikov metric is scale-invariant. This is just another reflection of the fact that the fermionic logarithmic operators do not themselves lead to any logarithmic divergences and act to cure the bosonic string theory of its instabilities. In fact, this property on its own is motivation for the identification (3.8) of worldsheet and target space regularization parameters which was used to arrive at the logarithmic conformal algebra. In turn, this correlation is then also consistent with the Galilean non-invariance (5.4) which derives from the twisted superparticle interpretation of the previous section. Nevertheless, the vanishing correlation functions in (2.4) and (2.8) indicate the existence of a hidden supersymmetry in the dynamics of moving D-branes. For instance, it is straightforward to check that the fermionic Noether supercurrents associated with spatial translations induce the same logarithmic scaling violations that the bosonic ones do [1, 2].

It is curious to note that the Zamolodchikov metric (7.2) becomes degenerate in the conformal limit $\Delta\to 0$, which corresponds to the infrared fixed point of the worldsheet field theory (in the sense that the size of the worldsheet is infinite in units of the ultraviolet cutoff). In this limit all two-point correlation functions involving the fermionic field χ_C vanish. Whether or not this implies that χ_C completely decouples from the theory requires knowledge of higher order correlators of the theory. Furthermore, in that case there are no logarithmic scaling violations, since $\langle\chi_D(z)\,\chi_D(w)\rangle = 2\xi/(z-w)$ in the limit $\Delta\to 0$. Generally, the vanishing of two-point functions in a logarithmic conformal field theory implies some special properties of the model. In the purely bosonic cases, it is known that such a vanishing property is associated with the existence of hidden symmetries corresponding to some conserved current [4]. A similar situation may occur in the supersymmetric case, indicating the presence of some new fermionic symmetries. For this to be case, there must be some other field to which the field χ_C couples. While the extra hidden symmetry may be related to the fact that χ_C is a null field in the subspace of primary fields, it should not be a true null field. This interesting issue deserves further investigation. Notice however that in the recoil problem, the pertinent correlation functions are non-vanishing in the slightly-marginal case where $\epsilon \neq 0$.

Notice also that the degeneracy of the metric (7.2) in the limit $\Delta \to 0$ may not be a true singularity of the moduli space of coupling constants. To further elaborate on this point requires computation of the corresponding curvature tensor, and its associated invariants which, being invariant under changes of renormalization group scheme, contain the true physical information of the theory. However, this again requires knowledge of the three-point and four-point correlation functions among the pertinent vertex operators, which at present are not available.

The Zamolodchikov metric is also a very important ingredient in the construction of the effective target space action of the theory. In the bosonic case such a moduli space action reproduces the Born-Infeld action for the D-brane dynamics in the Neumann representation [7]. The supersymmetrization of the worldsheet theory along the lines of section 4 will produce the same Born-Infeld action, with the only effect that the tachyonic instabilities are again removed and no renormalization of the coupling constants are required [24]. This is immediate due to the form of (7.2). On the other hand, the target space supersymmetrization of the Born-Infeld action in ten dimensions is known [24]. The photino field λ corresponds to the Goldstino particle of the super-Poincaré symmetry which is spontaneously broken by the presence of the D-brane. The resulting action does however possess local spacetime κ-symmetry. We may then expect an appropriate version of this action to emerge within the target space formalism of the previous section, with corresponding breaking of the fermionic κ-symmetry.

Acknowledgments

R.J.S. thanks J. Figueroa-O'Farrill for helpful discussions. The work of R.J.S. was supported in part by an Advanced Fellowship from the Particle Physics and Astronomy Research Council (U.K.).

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